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EXPERIMENTS WITH WAVES ON RELATIVISTIC ELECTRON BEAMS.(U)

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EXPERIMENTS WITH WAVES ON RELATIVISTIC ELECTRON BEAMS

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Final Report

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Submitted by

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January 1, 1981

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I. OBJECTIVE OF RESEARCH

This research has been directed towards a basic understanding of small-signal gain mechanisms in a free-electron laser (FEL).^{*} The experiments five years ago at Stanford University, in which 10 μm radiation was amplified on a 24 MeV electron beam have been explained in detail using single-particle models.¹ Experiments at Columbia University reported in 1978² in which 400 μm radiation was generated have been explained using a collective model. For a cold electron beam, the full comprehensive theory³ indicates that these mechanisms are not necessarily separable, so that the full description may be indispensable. Moreover the elementary theories often omit the effects of momentum spread on the electron beam, input coupling loss due to multi-mode propagation on the beam, and non-ideal electron orbits due to the actual magnetic fields of the wiggler with the guide field. For a beam with finite momentum spread, a geometric optics collective theory⁴ indicated yet another gain mechanism, originating with a wave-particle resonance. Furthermore, as shall be described in detail below, the presence of the axial guide field introduces additional complexity by permitting a large enhancement in gain if certain resonance conditions are fulfilled.

It is clear that careful experiments are required to sort out all these competing gain mechanisms. Our objective in this research therefore has been to assemble an apparatus to allow gain measurements to be performed on a FEL with detailed independently measured knowledge of both the equilibrium electron orbits in the wiggler and the momentum spread on the electron

* Many types of so-called free electron lasers have been discussed. In this work we restrict ourselves to a configuration in which a relativistic electron beam interacts with electromagnetic radiation while moving under the influence of a stationary periodic magnetic field and a uniform axial magnetic field.

beam. We furthermore have intended to study methods of altering the beam momentum distribution and beam density, so as to explore the parameter regimes where the limiting-case gain mechanisms may be identified.

In the first year of this work, good progress has been made towards these goals, as will be detailed below. In addition, important theoretical advances have been made in understanding the very complex orbits which electrons follow in a wiggler with an axial guide magnetic field. A small-signal gain theory has been developed which takes into account this axial guide field, and which indicates that large enhancements in gain may be realized if the parameters allow resonance between the cyclotron and undulatory motions.

We anticipate continuation of this research under joint AFOSR/ONR support.

II. RESULTS

Most of the substantive results achieved to date have either already been published, are in the process of preparation for publication, and/or have been presented at Scientific Conferences. We shall limit the detail presented here therefore, and refer the reader to the archival works themselves.

A. Experimental

Installation of the Febetron electron accelerator, vacuum chamber, axial guide magnetic field, and pulsed magnetic wiggler is complete. Diode current and voltage diagnostics have been developed. Cathode designs are under development to produce low current (100's of amperes) beams of low emittance. A novel 90° deflection momentum analyzer has been designed to determine the momentum spread for a sample of the beam. Low current cw

beams in the energy range 4-20 keV were employed to determine electron orbits in the wiggler/guide field combination. Beam analyzers were designed and employed to distinguish orbit deflection and helicity. A summary of one result obtained in this experiment is shown in Fig. 1, where the measured threshold for transition between helical and non-helical orbits has been plotted versus a universal parameter which embodies beam energy and guide field. Predictions of the idealized theory are also shown. Considering the approximation in the theory, we take the agreement to be remarkable. To our knowledge this is the first attempt to measure orbit dynamics in a FEL. This work was stimulated by theory performed by our collaborator L. Friedland [Phys. Fluids 23, 2376 (1980)]. Since the theory showed the stability threshold to persist for weakly-relativistic beams, we were able to perform the experiment at low energy. This work was presented recently at a conference [P. Avivi, F. Dotan, A. Fruchtmann, A. Ljudmirsky, and J. L. Hirshfield, Bull. APS 25, 910 (1980)]. This abstract is appended. The work is currently in draft form in preparation for publication.

B. Theoretical

One result was the discovery of a synergism between the undulatory motion induced by the wiggler, and the helical motion intrinsic to the orbit in the uniform guide field. Together these combine to enhance FEL gain. This work is now published [L. Friedland and J. L. Hirshfield, Phys. Rev. Lett. 44, 1456 (1980)]; a reprint is appended.

A second result is the extension of the small-signal gain theory published earlier [I. B. Bernstein and J. L. Hirshfield, Phys. Rev. A 20, 1661 (1979)] to include finite axial momentum spread. A representative result is shown in Fig. 2, where gain values for a FEL modeled using the

parameters of our experiment are used. The gain spectra for a cold beam, and for a beam with a parallel momentum spread between 5-20% is shown. This work was reported at a recent conference [A. Fruchtman and J. L. Hirshfield, Bull. APS 25, 911 (1980)]. The abstract is appended. The work is currently in manuscript form in preparation for publication.

A third result is concerned with a description of the non-helical orbits in a wiggler in a uniform axial magnetic field. For the "idealized" wiggler field $B_w = (\hat{e}_x \cos k_o z + \hat{e}_y \sin k_o z)B_w$, Friedland has computed orbits numerically. But S. Y. Park at Yale has shown that these orbits can be determined as well from an analytic theory; this allows greater physical insight to this complex problem than does the numerical approach. This analytic theory is in preparation for publication. It appears, however, that the strongly non-helical orbits may entail off-axis departures which are so large, as to call into question the validity of the idealized field. To examine this question, R. A. Smith and S. Y. Park at Yale have derived the exact field of a bi-filar helix. One objective is to assess the errors inherent in using the idealized field for the non-helical orbits. The ultimate objective of this work is a calculation of the spontaneous emission from electrons with strongly non-helical orbits. It appears that this radiation is rich in harmonics of the basic undulatory frequency; thus one might anticipate devising a FEL working on one or more of these "space harmonics", and thus furnishing shorter wavelength radiation than a conventional FEL with the same parameters.

Continuation of this research for at least two more years is anticipated by joint AFOSR/ONR support through ONR Contract N00014-79-C-0588, which was established originally on June 1, 1979 to support FEL theory. Co-Principal Investigators of the combined experimental/theoretical program are J. L. Hirshfield and I. B. Bernstein.

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FIGURE CAPTIONS

FIG. 1. Threshold values of B_w/B_z for onset of orbit instability, as a function of the universal parameter for a helical wiggler. Data points are shown for beam energies between 4-14 keV. The inset illustrates one of the analyzers used to determine helicity.

FIG. 2. Theoretical results for FEL power gain for the planned experiment, showing the effect of electron momentum spread.

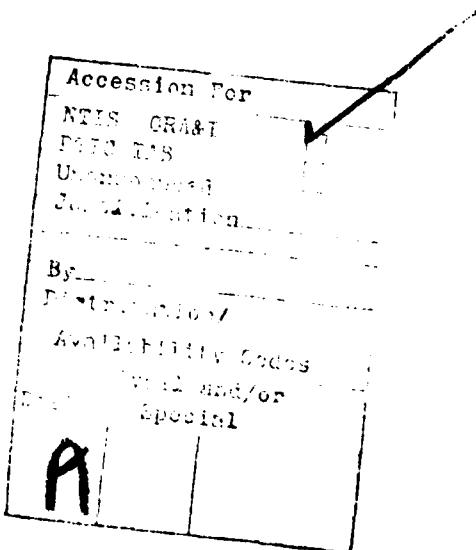
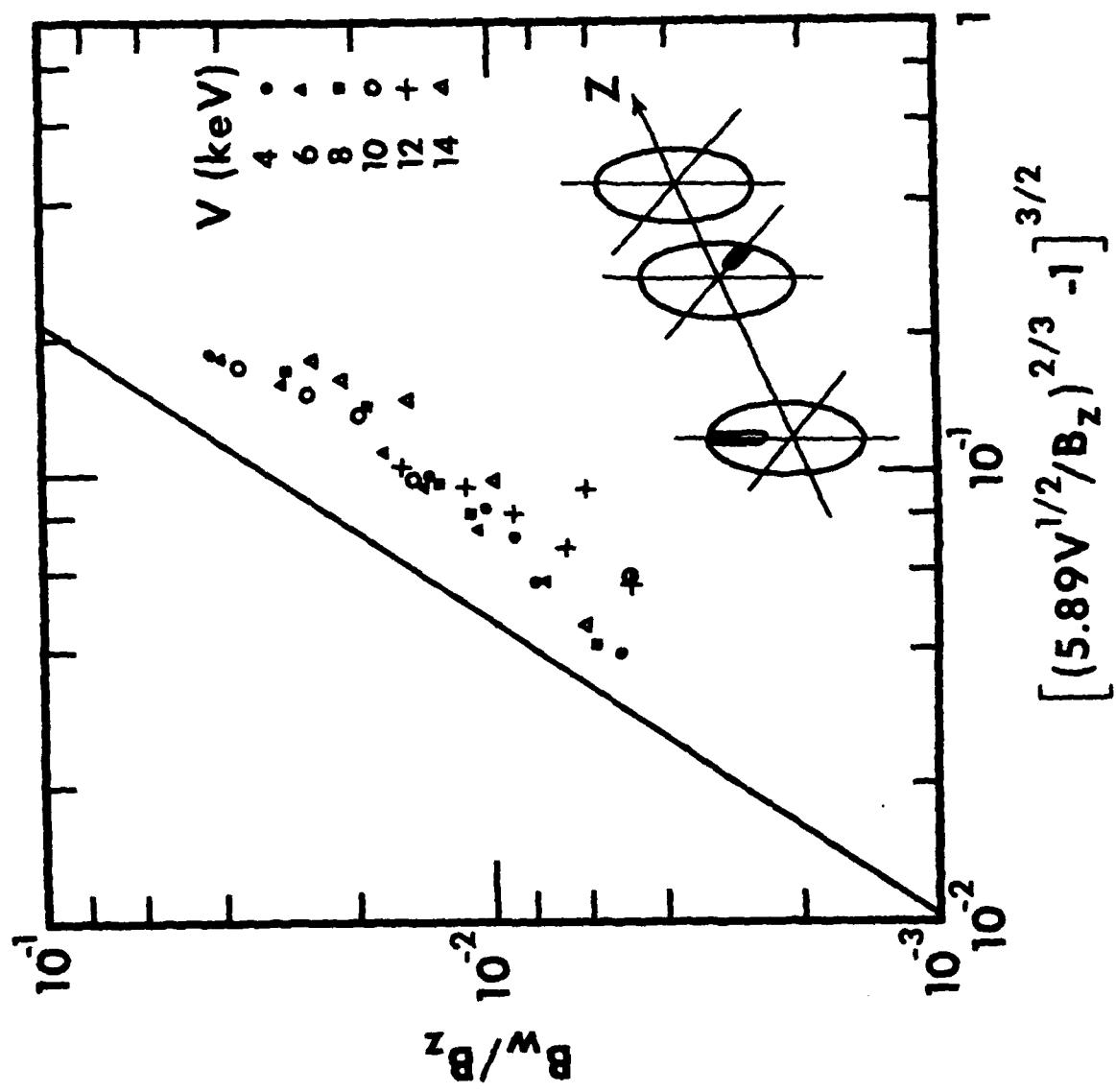


FIG. 1



FEL Amplifier Using Febeutron Accelerator

$$\gamma = 1.78$$

$$\lambda_0 = 3.6 \text{ cm}$$

$$s = 10$$

$$\xi = 0.2$$

$$J = 100 \text{ A/cm}^2$$

$$\frac{\Delta U}{U} = 0$$

0.05

0.10

0.15

0.20

Frequency (GHz)

40
38
36
34
32
30
28
26

32
30
28
26

28
26

40
38
36
34
32
30
28
26

FIG. 2

Free-Electron Laser with a Strong Axial Magnetic Field

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A small-signal theory is given for gain in a free-electron laser comprising a cold relativistic electron beam in a helical periodic transverse, and a strong uniform axial, magnetic field. Exact finite-amplitude, steady-state helical orbits are included. If perturbed, these orbits oscillate about equilibrium, so that substantial gain enhancement can occur if the electromagnetic perturbations resonate with these oscillations. This gain enhancement need not be at the cost of frequency upshift.

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Intensive activity is underway to exploit the gain properties of a relativistic electron beam undulating in a periodic transverse magnetic field. Such free-electron laser (FEL) configurations have provided oscillation at 3.4 (Ref. 1) and 400 μm^2 and amplification at 10.6 μm .² Theory has advanced apace,³ and elaborate schemes have been proposed for obtaining high FEL efficiency.⁵ A factor which limits the practical application of this interaction at wavelengths shorter than perhaps a few microns is the rapid decrease in small-signal gain G_0 as the electron energy increases. This is shown explicitly in the well-

known result⁶ for G_0 in the single-particle limit (i.e., when collective effects are negligible)

$$G_0 = (\omega_p \xi / k_0 c)^2 (k_0 L / 2\gamma)^3 F'(\theta). \quad (1)$$

Here ω_p and γ are the beam plasma frequency $N e^2 / m \epsilon_0$ and normalized energy W / mc^2 , k_0 and ξ are the helical transverse magnetic field wave number $2\pi / l$ and normalized strength eB_\perp / mck_0 , L is the interaction length, and $F'(\theta) = (d/d\theta)(\sin\theta / \theta)^2$ is the line-shape factor, with $\theta = [k_0 v_{z0} - \omega(1 - v_{z0}/c)](L/2c)$, where v_{z0} is the unperturbed electron axial velocity. The peak gain occurs at $\theta = 1.3$, where $F'(\theta) = 0.54$. For example, with γ

$\gamma = 10$, $l = 1.05$ cm, $\omega_p = 5 \times 10^7$ sec $^{-1}$, $\xi = 1$ ($B_\perp = 10.2$ kG), and $L = 130$ cm, the peak gain is $G_{op} = 0.00247$ at a wavelength of $105 \mu\text{m}$. For $\gamma = 100$, $l = 10.5$ cm, $\omega_p = 2 \times 10^9$ sec $^{-1}$, $\xi = 1$ ($B_\perp = 1.02$ kG), and $L = 260$ cm, the peak gain is $G_{op} = 0.00316$ at a wavelength of $10.5 \mu\text{m}$. These gain values may be large enough to sustain oscillations if highly reflecting mirrors are judiciously added but the strong helical fields required (particularly the 10.2-kG case) may be beyond the capability of present superconducting coil technology.⁷

A suggestion has appeared for enhancing the small-signal gain above values given by Eq. (1) (or for achieving comparable gains with smaller B_\perp) by employing a strong axial magnetic field so as to exploit resonance between the cyclotron frequency and the undulatory frequency.⁸ The present Letter presents a single-particle derivation for the small-signal gain of a FEL in a uniform axial magnetic field B_0 . We shall demonstrate that careful adjustment of the system parameters will allow enhancement of the FEL small-signal gain by an order of magnitude or more (for the above examples) without increasing the undulatory velocity. This result goes beyond that predicted by Sprangle and Granatstein⁶ who have suggested that the only effect of the axial magnetic field would be to add a multiplicative factor $(1 - \Omega/k_0 c \gamma)^{-2}$ to Eq. (1), due to the aforementioned resonance giving an enhanced undulatory velocity v_\perp , where $\Omega = eB_0/m$. This result is in fact predicted by our analysis as a limiting case. Of course, any mechanism which increases the undulatory velocity v_\perp would increase the gain, but this would also reduce the relativistic frequency upshift, since

$$\omega \approx k_0 c (1 - v_{30}/c)^{-1} = 2\gamma^2 k_0 c (1 + \gamma^2 v_\perp^2/c^2)^{-1}.$$

If, for example, $\gamma v_\perp/c = 1$ without the axial magnetic field, then a given gain enhancement η achieved through this resonance alone would result in a reduction in frequency upshift by a factor $(1 + \eta)/2$. The process we describe in this Letter will be shown to permit significant gain enhancement without undue sacrifice in frequency upshift. The gain enhancement originates when the electromagnetic perturbations resonate with the natural frequency of oscillation of electrons on finite amplitude equilibrium helical orbits. Prior workers have not considered this effect.

A full derivation of our result will be presented elsewhere.⁹ Exact unperturbed relativistic orbits are considered in the customary FEL model mag-

netic field

$$\vec{B}(\vec{r}) = B_0 \hat{e}_z + B_\perp (\hat{e}_x \cos k_0 z + \hat{e}_y \sin k_0 z). \quad (2)$$

These orbits, which have been the subject of recent study,¹⁰ can possess more than one steady state, depending upon γ , B_0 , B_\perp , and k_0 . These steady states are characterized by the normalized velocity components (i.e., $u_i = v_i/c$)

$$\begin{aligned} u_{10} &= 0, \quad u_{20} = k_0 \xi u_{30} / (k_0 u_{30} \gamma - \Omega/c), \\ u_{30} &= (1 - u_{20}^2 - \gamma^{-2})^{1/2}, \end{aligned} \quad (3)$$

where the basis vectors $\hat{e}_1(z) = -\hat{e}_x \sin k_0 z + \hat{e}_y \times \cos k_0 z$, $\hat{e}_2(z) = -\hat{e}_x \cos k_0 z - \hat{e}_y \sin k_0 z$, and $\hat{e}_3(z) = \hat{e}_z$ have been introduced to track the symmetry of the transverse magnetic field. Figure 1 shows u_{30} vs Ω/c for $k_0 = 6.0 \text{ cm}^{-1}$, $\xi = 1.0$, and $\gamma = 10$. For $\Omega > \Omega_{cr} \equiv k_0 c [(\gamma^2 - 1)^{1/3} - \xi^{2/3}]^{3/2}$, it is seen that only one branch exists (branch C). But for $\Omega < \Omega_{cr}$ two additional branches (A and B) are allowed: Branch B has been shown to be unstable, in that the orbits exhibit nonhelical, highly anharmonic motions, while branches A and C have orderly helical orbits. Stability is insured if $\mu^2 \equiv a^2 - bd > 0$, where $a = k_0 c u_{30} \xi / \gamma u_{20}$, $b = \Omega u_{20} / \gamma u_{30}$, and $d = k_0 c \xi / \gamma$. The quantity μ is the natural resonance frequency in response to small perturbations of the orbit: We shall show that strong resonance response of the electrons to electromagnetic perturbation can lead to enhanced FEL gain for small μ , i.e., for Ω close to Ω_{cr} .

The derivation of FEL gain proceeds by solving the single-particle equations of motion, subject to weak electromagnetic perturbing fields $\vec{E} = \hat{e}_x E_0 \cos(kz - \omega t)$ and $\vec{B} = \hat{e}_y (kc/\omega) E_0 \cos(kz - \omega t)$,

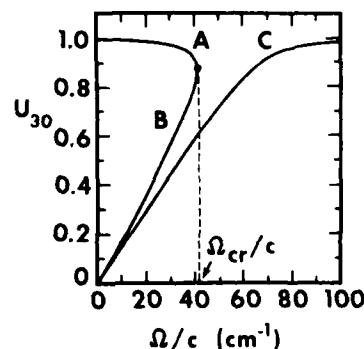


FIG. 1. Steady-state normalized axial velocity u_{30} as a function of normalized axial magnetic field Ω/c . For this example $k_0 = 6.0 \text{ cm}^{-1}$, $\xi = 1.0$, and $\gamma = 10$. Gain enhancement discussed in this work is for orbits on either branch A or branch C.

about the equilibrium orbits on either branch A or C as discussed above. These equations are

$$\dot{u}_1 = (k_0 c u_3 - \Omega/\gamma) u_2 - (k_0 c \xi/\gamma) u_3 - (\dot{\gamma}/\gamma) u_1 + (e E_1 / m c \gamma) (k c u_3 / \omega - 1), \quad (4)$$

$$\dot{u}_2 = - (k_0 c u_3) - \Omega/\gamma) u_1 - (\dot{\gamma}/\gamma) u_2 + (e E_2 / m c \gamma) (k c u_3 / \omega - 1), \quad (5)$$

$$\dot{u}_3 = (k_0 c \xi/\gamma) u_1 + (k c / \omega - u_3) (\dot{\gamma}/\gamma), \quad (6)$$

where $\dot{\gamma} = - (e/mc)(u_1 E_1 + u_2 E_2)$ and

$$2(E_2 + iE_1) = -E_0 \exp[i((k_0 + k)u_3 c t - \omega t + \alpha)]$$

with α the random initial electron phase. When time variations and electromagnetic fields are absent, Eqs. (4)–(6) lead to the exact steady states given by Eq. (3). To linearize Eqs. (4)–(6), we introduce the velocity perturbations $w_i = u_i - u_{i0}$ and retain only the lowest-order quantities. This results in $\ddot{w}_1 + \mu^2 w_1 = A E_0 \cos(\beta t + \alpha)$, or

$$w_1 = \frac{AE_0}{\mu^2 - \beta^2} [\cos(\beta t + \alpha) - \cos \mu t \cos \alpha + (\beta/\mu) \sin \mu t \sin \alpha] + \mu^{-1} \dot{w}_1(0) \sin \mu t, \quad (7)$$

where

$$A = (\alpha + \beta)(1 - u_{30}) + b u_{20}, \quad \beta = c(k + k_0)u_{30} - \omega, \quad \omega \approx kc, \quad \dot{w}_1(0) = (e E_0 / 2 \gamma m c)(1 - u_{30}) \sin \alpha,$$

and $w_1(0) = 0$. The other components follow from

$$\dot{w}_2 = -\alpha w_1 + (e E_0 / 2 m c \gamma)(1 - u_{30} - u_{20}^2) \cos(\beta t + \alpha), \quad w_2(0) = 0; \quad (8)$$

and

$$\dot{w}_3 = d w_1 + (e E_0 / 2 m c \gamma) u_{20}(1 - u_{30}) \cos(\beta t + \alpha), \quad w_3(0) = 0. \quad (9)$$

Equation (7) for w_1 exhibits the aforementioned natural resonance at frequency μ , while the electromagnetic perturbation drives the transverse motion at frequency β . Gain enhancement can be expected when μ is close to β .

The energy gain for an electron is calculated from $(mc/e)d\gamma/dt \approx -w_1 E_{10} - w_2 E_{20} - u_{20} E_{21}$. The first-order variation in electric field E_{21} originates from small phase variations as u_3 changes. Thus this becomes

$$(mc/e)d\gamma/dt = -w_1 E_{10} - w_2 E_{20} - \frac{1}{2} E_0 (k + k_0) c u_{20} \sin(\beta t + \alpha) \int_0^t dt' w_3(t'). \quad (10)$$

The third term in Eq. (10) is much larger than the other two on account of the factor $k + k_0$. The dominant single-particle energy transfer in the FEL (even with an axial magnetic field) is seen to be by work $e c u_2 E_2$ done along the transverse undulatory motion, enhanced by the strong variation in E_2 as its phase varies through w_3 . The energy variation [Eq. (10)] is averaged over random phase α to give $\langle d\gamma/dt \rangle$, which in turn leads to the gain through $G = 2(e_0 E_0)^2 N m c^2 \int_0^T dt \langle d\gamma/dt \rangle$, where N is the beam electron density and $T = L/c$ is the total interaction time for the electrons in a system of length L .

The final result is

$$G = \frac{\omega_p^2 k_0 c}{16\gamma} u_{20}^2 T^3 \left\{ \left[1 + \frac{a}{\mu_2} \left(\alpha + \beta + \frac{u_{20} b}{1 - u_{30}} \right) \right] \left[F'(\theta) - \frac{F(\theta + \varphi) - F(\theta - \varphi)}{2\varphi} \right] \right. \\ \left. + \frac{F(\theta + \varphi) - F(\theta - \varphi)}{2\varphi} - \frac{a}{\mu^2 T} \left[P'(\theta) - \frac{P(\theta + \varphi) - P(\theta - \varphi)}{2\varphi} \right] \right\}, \quad (11)$$

where $\theta = \beta T/2$, $\varphi = \mu T/2$, $F(x) = (\sin x/x)^2$, and $P(x) = x F(x)/2$; and where we have approximated $(k + k_0)(1 - u_{30}) \approx k_0$. We shall examine Eq. (11) in several limits.

For $\mu \gg \beta$, only the terms involving $F'(\theta)$ and $P'(\theta)$ in Eq. (11) are significant, and on branch A the latter of these is smaller than the former by at least a factor 2φ . Thus to a good approxi-

mation we may write

$$G(\mu \gg \beta) = Z(\omega_p^2 / 16\gamma) k_0 c u_{20}^2 T^3 F'(\theta), \quad (12)$$

where $Z = 2 + \mu^{-2} [a\beta + bd(1 - u_{30})^{-1}]$. In the case where the axial magnetic field is absent, $\Omega = 0$, $\mu = a = k_0 c u_{30} \gg \beta$, and $u_{20} = \xi/\gamma$. Thus, $Z \approx 2$ and Eq. (12) goes over to Eq. (1). When $\Omega \neq 0$ and μ

$\gg \beta$, gain enhancement can be achieved as claimed by the prior workers,⁸ due to resonant enhancement of u_{20} , but not without sacrificing frequency upshift, as discussed above.

However a more attractive possibility exists when μ is small, and approaches β . Here one can approximate $Z \approx \mu^{-2} bd(1 - u_{30})^{-1} \gg 1$; this results from resonance between the electromagnetic perturbation which gives oscillatory motion to the electron at a frequency β , close to its natural oscillation frequency μ . Gain enhancement due to large Z is seen to be possible without simultaneously increasing u_{20} , so that the desirable frequency upshift property of the FEL need not be sacrificed.

We define a gain enhancement factor $\eta = G/G_0$ to compare two free-electron lasers, identical except that one has a strong axial magnetic field, while the second does not. In the first laser, the transverse magnetic field B_\perp is reduced so that u_{20} is the same for both lasers. (This assures that both enjoy the same frequency upshift.) Then

$$\eta = Z \{ 1 - [F(\theta + \varphi) - F(\theta - \varphi)] / 2\varphi F'(\theta) \}. \quad (13)$$

We have evaluated Eq. (13) for two examples with the parameters cited in the first paragraph of this Letter, holding $|\theta| = 1.3$ where $|F'(\theta)|$ has its maximum value. The results are shown in Fig. 2 for the $\gamma = 10$ example. In Fig. 2(a) we plot the gain enhancement factor η as a function of the transverse magnetic field normalized strength ξ for the FEL with the axial guide magnetic field. The solid curves are for steady-state orbits on branch C; the dashed curves for branch A. On branch A, gain occurs for $\theta > 0$, while on branch C gain occurs for $\theta < 0$. Two transverse magnetic fields for the FEL without axial field corresponding to $\xi_0 = 1$ and 0.5 are shown. Figure 2(b) shows the required values of axial guide field. One sees a gain enhancement of 31 (on branch C) at $\xi = 5 \times 10^{-3}$ for an axial guide field of 102 kG. The transverse magnetic field required is reduced to 51 G, and the gain is increased to 0.0766 at $\lambda = 105 \mu\text{m}$. Higher gain is predicted on branch A. For the $\gamma = 100$ example at $\lambda = 10.5 \mu\text{m}$, we find a gain enhancement of 16 (on branch A) at $\xi = 3 \times 10^{-2}$ for an axial guide field of 99.5 kG. The transverse magnetic field required is reduced to 30.6 G, and the gain is increased to 0.0506.

Of course when the predicted single-pass gain is large (say > 0.1) this theory must be modified. Furthermore, finite electron momentum spread (neglected here) will mitigate against gain, as for a FEL without a guide field. These effects

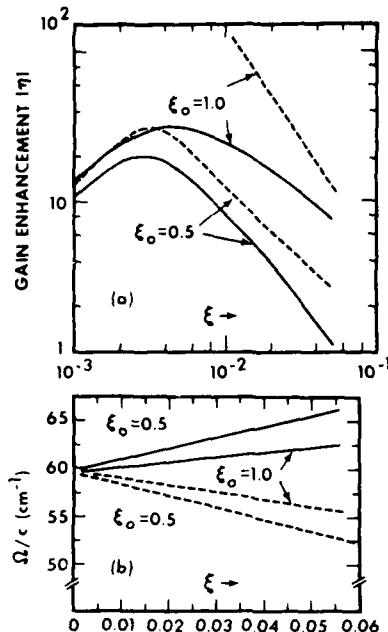


FIG. 2. (a) Gain enhancement $|\eta|$ and (b) corresponding normalized axial magnetic field Ω/c , vs transverse magnetic field parameter ξ . The values $\xi_0 = 0.5$ and 1.0 are for the FEL without axial field, and provide the same u_{20} as do the indicated (smaller) values of ξ for the FEL with the indicated axial field strength. Example is for $\gamma = 10$, $k_0 = 6.0 \text{ cm}^{-1}$, and $L = 130 \text{ cm}$. Solid curves, orbits on branch C; dashed curves, orbits on branch A. For high enhancement values, such as on the $\xi_0 = 1.0$ branch A example, the numerical precision required to compute accurate results suggests that the phenomenon is very sensitive to the system parameters.

deserve careful study. However, to the extent that these effects are negligible, our theory shows that provision of a strong uniform axial magnetic field can allow significant small-signal gain enhancement, and significant reduction in the required transverse magnetic field strength in a FEL, without undue compromise in operating frequency below that given by the idealized upshift value $2\gamma^2 k_0 c$.

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Abstract Submitted
For the Twenty-second Annual Meeting
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November 10 to 14, 1980

Subject Category Number 4.8 Microwave Generation

Experimental Study of Electron Orbit Stability in a FEL Magnetic Wiggler* P. AVIVI, F. DOTAN, A. FRUCHTMAN, A. LJUDMIRSKY, and J. L. HIRSHFIELD, ** Center for Plasma Physics, Hebrew University of Jerusalem, Israel, and L. FRIEDLAND, Yale Univ.--Free electron laser experiments are being widely pursued in which a relativistic electron beam propagates along the axis of a helical magnetic wiggler. An axial guide magnetic field is usually present for beam collimation, and a recent theory¹ shows how both steady-state and dynamical resonant effects due to this guide field can lead to gain enhancement. Theory for equilibrium orbits in this magnetic field configuration indicates that stable helical orbits are possible, but that strongly non-helical ("unstable") orbits are easily produced.² Results of an experiment are presented in which novel orbit analyzers were employed to search for the threshold for orbit stability, as a function of beam energy, guide field, and wiggler field. The observations are consistent with theory.

*Sponsored by AFOSR and ONR.

**Also Yale University.

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Abstract Submitted
 For the Twenty-second Annual Meeting
 Division of Plasma Physics
 November 10 to 14, 1980

Subject Category Number 4.8 Microwave Generation

Influence of Electron Energy Spread on Amplification in a Free Electron Laser* A. FRUCHTMAN and J. L. HIRSHFIELD**, Center for Plasma Physics, Hebrew Univ. of Jerusalem, Israel—A collective theory has been published¹ for small-signal amplification in a free electron laser comprising a relativistic electron beam propagating along the axis of a static helical pump magnetic field. However, detailed analysis has heretofore been limited to the case of a cold electron beam, i.e. $f_0(a, \beta, \gamma) = \text{const.} \delta(a) \delta(\beta) \delta(\gamma - \gamma_0)$, where a and β are the transverse components of the canonical angular momentum and γ is the total energy normalized to mc^2 . The present work extends the above analysis to the case of a beam with finite energy spread, i.e. $f_0(a, \beta, \gamma) = \text{const.} \delta(a) \delta(\beta) [H(\gamma - \gamma_1) - H(\gamma - \gamma_2)]$, where H is the step function. This model accounts for reductions in amplification due to phase mixing, but not enhancement of amplification due to wave-particle coupling.² Results will be shown for several cases of practical importance.

*Sponsored by AFOSR and ONR.

**Also Yale University.

¹I.B.Bernstein and J.L.Hirshfield, Phys.Rev.A 20, 1661 (1979).

²I.B.Bernstein and J.L.Hirshfield, Phys. Rev. A 20, 1661 (1978).

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Subject Category Number 4.8 Microwave Generation

Particle Orbits in a Magnetic Wiggler With a Uniform Guide Magnetic Field* R.A. SMITH and J.L. HIRSHFIELD, Yale Univ., and S.Y. PARK† and J.M. BAIRD‡, Naval Research Lab.—An exact analytic solution is given for the orbit of a charged particle moving in the combined magnetic field of a bifilar helical wiggler and a uniform solenoid. The axial velocity is shown to satisfy an anharmonic oscillator equation whose solutions are Jacobian elliptic functions. Other velocity components are readily determined from the axial velocity. Solutions are classified according to the type of Jacobian elliptic function, which in turn is determined by magnetic field parameters and initial particle conditions. The exact solution allows calculation of spectra of single-particle radiation, and allows formulation of the linearized Vlasov-Maxwell theory for an ensemble.

*Supported by AFOSR and ONR.

†B-K Dynamics, Rockville, MD.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Installation of the Febetron electron accelerator, vacuum chamber, axial guide magnetic field, and pulsed magnetic wiggler is complete. Diode current and voltage diagnostics have been developed. Cathode designs have begun to produce low current (100's) of amperes) beams of low emittance. A novel 90 degree deflection momentum analyzer has been designed to determine the momentum spread for a sample of the beam. Low current cw beams in the energy range 4-20 keV were employed to determine electron orbits in the wiggler/guide field combination. Beam analyzers were designed and employed to distinguish orbit deflection and helicity.			

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